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Modeling correlated market and credit risk in fixed income portfolios

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Abstract

Current risk assessment methodologies separate the analysis of market and credit risk and thus misestimate security and portfolio risk levels. We propose a new approach that relates financial market volatility to firm specific credit risk and integrates interest rate, interest rate spread, and foreign exchange rate risk into one overall fixed income portfolio risk assessment. Accounting for the correlation between these significant risk factors as well as portfolio diversification results in improved risk measurement and management. The methodology is shown to produce reasonable credit transition probabilities, prices for bonds with credit risk, and portfolio value-at-risk measures. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Risk assessment methodologies seek to assess the maximum potential change in the value of a portfolio with a given probability over a pre-set horizon

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resulting from changes in market factors, credit risk, and liquidity risk. The risk in owning a portfolio of risky fixed income securities is a function of changes in the risk-free term structure (interest rate risk), macroeconomic or market conditions which affect the overall risk premium of an asset class (spread risk), foreign exchange rates (FX risk), and the credit quality of the assets in the portfolio (credit risk). We will use the term market risk to refer to the aggregate impact of interest rate, interest rate spread, and FX risk.

The current practice is to undertake market and credit risk assessments separately. Combining such separate risk measures into one overall portfolio risk measure is not easily accomplished. The absence of reliable overall portfolio risk measures creates problems determining capital adequacy requirements, capital-at-risk measures, hedging strategies, etc.

Given the correlated nature of credit and market risk (Fridson et al., 1997), the importance of an integrated risk assessment methodology seems apparent. To address the above risk measurement problem we develop a diffusion-based methodology for assessing the value-at-risk (VaR) of a portfolio of fixed income securities with correlated interest rate, interest rate spread, exchange rate, and credit risk. This is accomplished by simultaneously simulating both the future financial environment in which financial instruments will be valued and the credit rating of specific firms. The fundamental basis of this methodology is the contingent claims analysis (CCA) proposed by Merton (1974) with a number of stochastic financial environment variables. Appropriately calibrated for the volatility of the period and firms to be studied the simulation methodology developed in this paper is shown to produce reasonable credit transition probabilities, valuations for bonds with credit risk, and portfolio VaR measures including the marginal impact of each risk factor. The model has the potential to be extended to undertake financial institution asset and liability risk assessments as well as financial system systemic risk assessments (see Barnhill et al., 2000).

Overall portfolio risk in this model is a function of six types of underlying correlated and uncorrelated stochastic variables including interest rates, interest rate spreads, FX rates, returns on equity market indices (i.e. systematic risk), firm specific equity returns (i.e. unsystematic risk), and default recovery rates. Given the number of significant variables and the complexity of the relationships a closed form analytical solution for portfolio VaR is not available. Therefore, we use a numerical simulation methodology.

As an overview, both the future financial environment in which the asset will be valued and the credit rating of specific firms are simulated. The financial environment is represented by eight correlated (approximately) arbitrage-free term structures of interest rates (United States Treasury, Aaa, ..., Caa–C), a single FX rate (e.g. Japanese yen), and a set of 24 equity market indices representing various sectors of the economy (in practice any number of term structures, FX rates, and equity indices could be simulated). The correlated evolution of the market value of the firm's equity, its debt ratio, and credit rating are then simulated in the context of the simulated financial environment. The structure of the methodology is to select a time step over which the stochastic variables are allowed to fluctuate in a correlated random process. The firm specific equity returns and security specific default recovery rates are assumed to be uncorrelated with each other and the other stochastic variables. For each simulation run a new financial environment (correlated interest rate term structures, FX rate, and market equity returns) as well as firm specific market value of equity, debt ratio, credit rating, and default recovery rates are created. This information allows the correlated values of financial assets to be estimated, and after a large number of simulations, a distribution of portfolio values is generated and analyzed. A similar methodology has been applied to portfolios containing mortgages, variable rate loans, other fixed income securities, equities, real estate, and derivative securities (Barnhill et al., 2000).

The paper is organized in the following manner. First, a review of current credit risk and market risk analysis methodologies is provided. Second, the model for integrating market and credit risk is developed as well as discussion of how the parameters necessary for the model are empirically estimated. Third, the simulated credit transitions for representative bonds are compared to historical transition matrixes, bond valuation tests are performed, and the simulation methodology is used to assess integrated credit and market risk for various portfolios. Simulated and historical portfolio risk analyses are compared. Finally the conclusions are given.

2. Credit risk analysis

Two general methodologies have been developed to price debt instruments subject to credit risk and in some cases correlated interest rate and credit risk. The contingent claims methodology models the asset value of the firm as a stochastic process and prices the debt as an option on the value of the firm (Merton, 1974). Longstaff and Schwartz (1995) extend this methodology to the pricing of debt instruments facing correlated interest rate and credit risk. In the diffusion models, the value of a firm's bond is a function of the underlying asset, the total firm value, the volatility of the firm's value, and the term structure. While this is a theoretically tractable methodology, it does not produce results consistent with the observed short-term credit spreads. This limitation led to the development of a second general methodology, referred to as reduced form models or hazard rate models (Jarrow et al., 1997; Das and Tufano, 1996; Madan and Unal, 1998; Duffie and Singleton, 1997, 1999). The reduced form models assume fixed probabilities for credit quality changes and a fixed recovery rate in the event of default. While these models produce more realistic short-term credit spreads, there is no underlying theoretical model driving bond prices.

Credit risk analysis assesses the impact of stochastic changes in credit quality (including default) on the value of a fixed income security or a portfolio of fixed income securities. This requires estimating the probability of financial assets migrating to different risk categories (bond rating) over a pre-set horizon. The values of the financial assets are then estimated for each possible future risk category using forward rates from the current term structure appropriate for each risk class. There are currently a number of different packages available to assess credit risk including CreditMetrics™, CreditRisk+™, Credit View, Loan Analysis System (LAS), and ValueCalc™ (Barnhill, 1998). Altman and Saunders (1998) develop an analytical model that relies on the Altman (1993) z'' score to determine the probability of default over time. Jarrow et al. (1997) develop a model based on historical transition probabilities, which follow a Markov process, to price bonds. This methodology can also be applied for credit risk assessment.

Jarrow et al. (1997) decomposes fixed income instruments into zero-coupon bonds. By assuming the bond is held to maturity, the authors collapse the problem of credit risk into only two states of nature, default or not default. If the payoff in default is known with certainty, the value of a zero-coupon risky bond can be represented in continuous time as

$$V_t = p_t(C_t \mathrm{e}^{-rt}) + (1 - p_t)(D_t \mathrm{e}^{-rt}), \tag{1}$$

where V_t is the value of the zero-coupon bond at time t, p_t the probability of not defaulting at time t, C_t the cash flow (principal repayment) on the zero-coupon bond at time t, r the interest (discount) rate at time t, and D_t the value of the bond in default at time t. Since p_t and D_t are assumed to be certain, the appropriate discount rate is the risk-free rate at time t.

2.1. Credit event

Credit risk is sometimes thought of as the probability of default. However, this definition of credit risk views the bond in only two states: defaulted or not defaulted. In the more complex setting necessary to price bonds that may be sold before maturity, credit risk is a continuum with multiple states with each state representing an associated probability of default. Hence, temporal credit risk is a function of the probability of a change in the value of the bond associated with a transition in the probability of default over time, and credit risk can be either a positive or negative shift. A positive credit change decreases the likelihood of the bond defaulting and is commonly related to an increase in the bond's rating and value, an upgrade. A negative credit event is related to either default or a downgrade, which can lead to a significant loss in the value of the bond. The significance of credit ratings is evident by the importance third parties and financial regulators place on them for assessing the risk of financial institutions, mutual funds, and pension funds.

Clearly some caution is in order when utilizing credit ratings. First, credit rating changes lag market pricing (Ederington et al., 1987). Second, credit ratings attempt to assess the overall credit risk of a fixed income security, and to do this they combine both the probability and severity of default into a single measure. This impedes the comparison of bonds across seniority classes and can lead to some confusion. For example, a senior secured Ba bond likely has a higher probability of default than a junior subordinated Ba bond. The senior secured bond has less credit risk in the event of default because on average it will have a higher recovery rate. Thus for the senior secured bond to be rated the same as the junior subordinated bond, the other component of credit risk, the probability of default, must be greater.

2.2. Credit risk

Changes in bond ratings reflect changes in the perceived ability of the firm to meet its financial obligations. Such credit quality changes may result from changes in macroeconomic conditions (systematic risk) or from changes in the unique financial condition of the firm (unsystematic risk). The correlated impacts of macroeconomic factors on the credit quality of many firms imply a correlation in credit risk across firms in various industries as well as a correlation between market and credit risk. The simulation methodology developed in this paper captures such correlations.

An analysis of the effect of a shift of one rating category on the value of a bond is provided in Table 1. Non-callable term structures, estimated for 12/31/98, are used in this example. The significance of a credit event on the value of a bond is apparent. This is especially true as credit rating declines. A credit migration from Aaa to Aa for a five-year bond decreases the value of the bond by 0.96% while a credit migration from B to Caa decreases the value of the bond by 16.53%. The comparison in Table 1 of the effect of credit migration between the five- and ten-year bonds, as well as the discrepancy of the change in the price based upon an upgrade versus a downgrade, demonstrates the effect of duration and convexity on credit risk.

2.3. Transition matrixes

To assess credit risk each possible credit transition must be associated with a probability. One method to project future transitions is to rely on historical transition probabilities (see Jarrow et al., 1997; CreditRisk+, 1997). Moody's Investor Service and Standard & Poor's are two of the most prominent firms that compile historical probabilities of credit transition by rating category. For this study, Moody's transition matrixes [1] are utilized for comparison to our

[1] See Carty and Lieberman (1996).

Table 1
Changes in bond values resulting from rating changes

Rating category	Aaa	Aa	A	Baa	Ba	B	Caa
Five-year bond yield	0.0509	0.0531	0.0555	0.0600	0.0879	0.1034	0.1501
Downgrade to next category	99.04	98.96	98.07	88.75	93.97	83.47	34.00
% change downgrade	−0.96	−1.04	−1.93	−11.25	−6.03	−16.53	−66.00
Upgrade to next category	n.a.	100.96	101.05	101.95	111.96	106.25	118.16
% change upgrade	n.a.	0.96	1.05	1.95	11.96	6.25	18.16
Ten-year bond yield	0.0543	0.0575	0.06	0.0649	0.0966	0.1143	0.21
Downgrade to next category	97.56	98.12	96.39	79.53	89.35	56.38	34.00
% change downgrade	−2.44	−1.88	−3.61	−20.47	−10.65	−43.62	−66.00
Upgrade to next category	n.a.	102.47	101.90	103.69	123.37	111.43	157.60
% change upgrade	n.a.	2.47	1.90	3.69	23.37	11.43	57.60

The change in the value of the bond is calculated by changing the required yield to maturity to that of the adjacent rating category. Bond yields to maturity are estimated as of 12/31/98. Bonds are assumed to initially be priced at PAR (100). For example, a five-year Ba bond priced at 100 would be priced at 93.97 if downgraded to B, or it would be priced at 111.96 if upgraded to Baa.

simulated transition probabilities. Given the Carty and Lieberman (1996) finding of no systematic bias in the withdrawn category between upgrades and downgrades, the transition matrixes given in Table 2 are adjusted to eliminate the withdrawn category.

Utilizing a historical transition matrix to assess credit risk has a number of problems. For example, Fridson et al. (1997) found a relation between macroeconomic conditions and default probability. Thus credit transition probabilities differ considerably during economic recession and expansion. In addition, we believe that to accurately assess overall financial risk a methodology must account for correlated market and credit risk across an entire portfolio of assets. This is difficult to achieve using historical transition matrixes. The method presented in this paper relates the value of a firm's equity and ultimately its credit rating systematically to the simulated returns on equity indices for various sectors of the economy. In this way credit transition probabilities are systematically related to economic expansion and contraction as well as being correlated with changes in other financial environmental variables (e.g. interest rates, FX rates, etc.).

2.4. Recovery rates in the event of default

In the case of default, the distribution of recovery rates must also be modeled. Carty and Lieberman (1996) and Altman and Kishore (1996) conclude that average recovery rates increase with the seniority and security of the bonds. However, within a seniority class there is a wide distribution of realized recoveries. Additionally, Altman and Kishore (1996) found some indication that recovery rates may be a function of industry. Given the large standard

Table 2
Moody's transition matrixes adjusted for withdrawn ratings (1920–1996)

Initial rating	Aaa	Aa	A	Baa	Ba	B	Caa–C	Default
Probability of rating after one year								
Aaa	92.28%	6.43%	1.03%	0.24%	0.02%	0.00%	0.00%	0.00%
Aa	1.28%	91.68%	6.09%	0.70%	0.17%	0.02%	0.00%	0.06%
A	0.07%	2.45%	91.59%	4.97%	0.67%	0.11%	0.02%	0.13%
Baa	0.03%	0.26%	4.19%	89.41%	5.07%	0.66%	0.07%	0.30%
Ba	0.01%	0.09%	0.43%	5.09%	87.23%	5.47%	0.45%	1.23%
B	0.00%	0.04%	0.15%	0.67%	6.47%	85.32%	3.44%	3.90%
Caa–C	0.00%	0.02%	0.04%	0.37%	1.38%	5.80%	78.78%	13.60%
Probability of rating after three years								
Aaa	81.64%	13.93%	3.26%	0.75%	0.36%	0.02%	0.00%	0.03%
Aa	3.09%	78.67%	14.54%	2.53%	0.76%	0.09%	0.02%	0.29%
A	0.18%	5.80%	80.42%	10.26%	2.19%	0.45%	0.07%	0.63%
Baa	0.08%	0.76%	10.26%	75.43%	9.55%	2.12%	0.26%	1.54%
Ba	0.05%	0.25%	1.62%	12.14%	69.19%	10.59%	1.44%	4.72%
B	0.01%	0.10%	0.44%	2.26%	13.67%	65.88%	5.60%	12.04%
Caa–C	0.00%	0.00%	0.03%	1.04%	3.88%	10.12%	56.79%	28.14%

To examine if credit transitions are Markov and as benchmark for the transition probabilities generated using a CCA, Moody's historical transition probabilities are reported (Carty and Lieberman, 1996). Carty and Lieberman find no bias in the withdrawn category. Thus, the transition probabilities are adjusted for bonds that have had their ratings withdrawn by Moody's.

deviation of realized recovery rates, [2] in our proposed simulation the default recovery rate is modeled as a stochastic variable drawn from a beta distribution, which allows the recovery rate to fall within 0% and 100% while maintaining an assumed mean and standard deviation.

2.5. Utilizing transition matrixes and recovery rates to value bonds before maturity

Table 3 gives an example of a standard credit risk calculation for a ten-year B-rated bond trading with an initial PAR value of $1000. The value of the cash flows from the bond (price of the bond at $t = 1$ plus the coupon payment) is calculated at a one-year time step assuming the implied forward rates from the current term structure are the actual arrived at spot rates. The distribution of possible values multiplied by the probability of arriving at that credit quality is the mean expected value of the bond at the end of one year, $1,054.66 in this example. The standard deviation of the bond's value at the end of one year, $174.12, can then be easily calculated. Confidence levels can also be calculated

[2] See Carty and Lieberman (1996) and Altman and Kishore (1996).

Table 3
Credit risk analysis for a ten-year B-rated bond

	Probability of transition (%)	Coupon	Bond value $t = 1$	Bond plus coupon value $t = 1$	Prob. weighted	Change from mean
Aaa	0.00	$117.61	$1,432.45	$1,550.06	$-	$495.39
Aa	0.04	$117.61	$1,400.63	$1,518.23	$0.61	$463.57
A	0.15	$117.61	$1,377.47	$1,495.07	$2.24	$440.41
Baa	0.67	$117.61	$1,333.98	$1,451.59	$9.73	$396.92
Ba	6.47	$117.61	$1,084.28	$1,201.89	$77.76	$147.22
B	85.32	$117.61	$972.12	$1,089.73	$929.76	$35.06
Caa	3.44	$117.61	$501.89	$619.50	$21.31	$(435.17)
Default	3.90	–	–	340	$13.26	$(714.66)
Average					$1,054.66	
Std. dev.					$174.12	
99% confidence level					$340.00	
95% confidence level					$619.50	

A sample of a standard credit risk analysis for a B-rated bond with a ten-year maturity with an initial PAR value of 1000 is provided. The probability transitions are from Moody's one-year transition matrix. The spot and implied forward rates are estimated from the 12/31/98 yield curve. The cash flows from the bond (price + coupon) are revalued at the end of the first year utilizing the implied forward rates as of 12/31/98. Since the yield curve is upward sloping in this example, the value of the bond at the end of the first year is worth less than its original value even if the bond stays in the same rating category.

in this framework by determining the level at which a cumulative percentage exceeds the confidence level. The cumulative percentage exceeds 95% when the bond is rated Caa ($619.50) and 99% when the bond is in default ($340.00). Similar to Jarrow et al. (1997), the analysis given in Table 3 assumes that the credit transition probabilities and recovery rate in default are deterministic.

3. An integrated model of correlated market and credit risk

In this section, the simulation model for estimating correlated market and credit risk is developed. We believe it is necessary to simultaneously simulate the future financial environment in which bonds will be valued and the correlated evolution of the credit quality of the financial instruments to fully evaluate the risk characteristics of instruments and portfolios. This model is a modification and extension of the diffusion models developed by Merton (1974) and Longstaff and Schwartz (1995), applied to a multi-asset portfolio.

The price of a fixed income security is a function of the term structure for that asset. For current demonstration purposes, we have eight mutually exclusive asset classes (Aaa,. . ., Default) into which a bond may fall. The term structures for each asset class (excluding the default category) is a stochastic variable.

The simulation of bond credit rating is undertaken in a reduced form of the CCA framework. As developed by Black and Scholes (1973) and more explicitly by Merton (1974) the firm's stockholders hold a call option on the firm and the debt ratio is a measure of how far the call option is in the money. In addition to a number of standard efficient market assumptions the CCA framework assumes that the dynamics for the value of the firm, V, through time can be described by a diffusion-type stochastic process with the stochastic differential equation

$$\mathrm{d}V = (\alpha V - C)\,\mathrm{d}t + \sigma V\,\mathrm{d}z, \tag{2}$$

where α is the instantaneous expected rate of return on the firm per unit time, C the total dollar payout by the firm per unit of time to either its shareholders or liabilities holders, σ^2 the instantaneous variance of return on the firm per unit of time, and $\mathrm{d}z$ a standard Gauss–Wiener process.

We relax and modify some of the standard assumptions found in the CCA framework and make some additional assumptions as follows:

Assumption 1. The value of debt in the debt ratio refers to the face value of the debt, which is the cash flow due at maturity of the bond.

Assumption 2. The default-free interest rate, interest rate spreads, equity indices, and FX rates are correlated stochastic variables.

Assumption 3. The firm's debt ratio (D/V) and volatility (σ) can be used to determine the appropriate risky term structure (AAA,..., Default) to value the bond's cash flows.

Assumption 4. If the bond defaults, the recovery rate is stochastic and drawn from a beta distribution with a known mean (e.g. 34%) and standard deviation (e.g. 25%) (see Altman and Kishore, 1996).

Assumption 5. The firm's expected return on equity and firm specific equity return volatility can be estimated using a one factor CAPM model (multi-factor models would also be feasible).

Assumption 6. The expected growth rate in the market value of the firm's common stock is equal to the firm's expected return on equity minus its dividend yield.

Assumption 7. The dividend yield is constant over the time period simulated.

Assumption 8. The firm has an expected growth rate in assets and a target debt ratio that are constant.

Our goal is to model the stochastic changes in the market value of a bond. The factors that cause stochastic shifts in a bond's price are correlated interest rate, interest rate spread, exchange rate, and credit rating changes (including default). Default risk refers to the ability of the firm to meet set cash payments, which is in reference to the face value of the debt (book value), and the default recovery rate if the payments are not met. Work by Ogden (1987) and Barnhill and Maxwell (1998) suggests that Assumption 3 is reasonable as debt ratios can be used to reasonably map bond ratings if the industry specific nature of business risk is taken into account. Given Assumption 8, we model the firm has having a fixed financing plan (i.e. equity and debt sales or repurchases) over the simulation period. Thus variations in the debt to value ratio and credit rating at time step Δt reflect changes in the market value of the firm's equity. This is consistent with the findings that stock returns lead bond returns in reflecting firm specific information over a short-term horizon (Kwan, 1996) and over a longer-term horizon (Gebhardt, 1999).

3.1. Simulating stochastic term structures

For this study, the Hull and White extended Vasicek model (Hull and White, 1990, 1993, 1994) is used to model stochastic risk-free (e.g. U.S. Treasury) interest rates. In this model interest rates are assumed to follow a mean-reversion process with a time dependent reversion level. The simulation model is robust to the use of other interest rate models.

The model for r is

$$\Delta r = a\left(\frac{\theta(t)}{a} - r\right)\Delta t + \sigma\,\Delta z, \tag{3}$$

where Δr is the risk-neutral process by which r changes, a the rate at which r reverts to its long-term mean, r the instantaneous continuously compounded short-term interest rate, and $\theta(t)$ an unknown function of time which is chosen so that the model is consistent with the initial term structure and is calculated from the initial term structure as

$$\theta(t) = F_t(0,t) + \alpha F(0,t) + \frac{\sigma^2}{2a}(1 - \mathrm{e}^{-2at}).$$

$F(0,t)$ is the forward interest rate at time t as calculated at time 0, $F_t(0,t)$ the derivative of the forward interest rate with respect to time, Δt a small increment to time, σ the instantaneous standard deviation of r, which is assumed to be constant, and Δz a Wiener process driving term structure movements with Δr being related to Δt by the function $\Delta z = \varepsilon\sqrt{\Delta t}$.

Table 4
Term structure by bond rating class and mean reversion and volatility of term structures by bond rating class

		Asset class							
		Treasury	Aaa	Aa	A	Baa	Ba	B	Caa
Term structure information: 12/31/98									
Time to maturity	1	4.59%	4.96%	5.00%	5.17%	5.53%	7.41%	8.78%	12.00%
	5	4.39%	5.09%	5.31%	5.55%	6.00%	8.79%	10.43%	15.01%
	10	4.59%	5.43%	5.75%	6.00%	6.49%	9.66%	11.43%	21.00%
	15	4.89%	5.80%	6.18%	6.43%	6.95%	10.12%	11.99%	21.00%
Term structure parameter estimates (empirically estimated from 1/93–12/98)									
Mean-reversion rate		0.048	0.061	0.062	0.058	0.084	0.171	0.069	0.142[a]
Std. dev. of the short interest rate		0.007	0.010	0.010	0.010	0.011	0.014	0.010	0.039[a]
Std. dev. of the interest rate spread (e.g. Ba–Baa)		n.a.	0.002	0.002	0.001	0.002	0.011	0.011	0.034[a]

The term structure is estimated from Standard & Poor's *CreditWeek* and the Lehman Brothers bond database. Mean-reversion rates and volatilities of the short rates are estimated empirically over the January 1993 to December 1998 time period.
[a] For Caa–C.

The above mean-reversion and volatility rates can be estimated from a time series of short-term interest rates or implied from cap and floor prices. In this study they are estimated from a time series of short-term interest rates over the 1993–1998 period (Table 4). Given a simulated future value of r, the initial term structure, and the other parameters of the model a complete term structure of risk-free interest rates can be calculated and financial assets can be re-valued at time step Δt.

Once the risk-free term structure has been estimated then the Aaa term structure is modeled as a stochastic lognormal spread over risk free, the Aa term structure is modeled as a stochastic spread over Aaa, etc. The mean value of these simulated credit spreads are set approximately equal to the forward rates implied by the initial term structures for various credit qualities (e.g. Aaa). This procedure insures that all simulated credit spreads are always positive and that the simulated term structures are approximately arbitrage-free.

The first step in modeling the eight different term structures is to determine the appropriate initial yield curves. For this study term structure estimates for United States Treasury securities, Aaa, Aa, A, Baa, Ba, and B bonds are taken from Standard & Poor's *CreditWeek*, while the Caa term structure is estimated from the Lehman Brothers bond database (Table 4). In addition a time series of short-term yields for the various credit ratings is estimated for 1993–1998. This time series is used to estimate the volatility of the various credit spreads (e.g. Aa vs. Aaa,..., B vs. Ba, etc.). Table 4 gives the estimated volatilities for the various interest rate spreads.

3.2. Simulating asset returns

The model utilized to simulate the value of the equity market indices and FX rate (S) assumes that S follows a geometric Brownian motion where the expected growth rate m and volatility σ are constant (Hull, 1997, p. 362). The expected growth rate is equal to the expected return on the asset μ minus its dividend yield q. For a discrete time step, Δt, it can be shown that

$$S + \Delta S = S \exp\left[\left(m - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}\right]. \tag{4}$$

ε is the random sample from a standardized normal distribution.

The return on the market index (K_{m}) is estimated as

$$K_{\mathrm{m}} = ((S + \Delta S)/S) + q. \tag{5}$$

The return on equity for individual firms is simulated using a one-factor model:

$$K_i = R_{\mathrm{F}} + \mathrm{Beta}_i(K_{\mathrm{m}} - R_{\mathrm{F}}) + \sigma_i \Delta z, \tag{6}$$

where K_i is the return on equity for the firm i, R_{F} the risk-free interest rate, Beta_i the systematic risk of firm i, K_{m} the simulated return on the equity index from Eq. (5), σ_i the firm specific volatility in return on equity, and Δz a Wiener process with Δz being related to Δt by the function $\Delta z = \varepsilon\sqrt{\Delta t}$.

In the simulations where bonds are priced in a risk neutral framework the expected return on the equity index is set equal to the risk-free rate. In the simulations undertaking integrated market and credit risk analysis on portfolios of bonds the expected return on the equity indices is set equal to the risk-free rate plus a long-term average risk premium of 8%. The average dividend yield on the S&P500 from 1993 to 1998 of approximately 2.6% (source: DRI) is used as the market dividend yield. The 1998 equity return volatility for the S&P500 of 23 percent is utilized as the estimate for market volatility for all equity indices. The volatility of the yen versus U.S. dollar FX rate is assumed to be its 1987–1996 average of 10%.

After simulating the market return, the return on equity for an individual firm is estimated in the CAPM framework (Eq. (6)). The first step in calculating the expected return on equity for a "typical" firm in a particular rating class (e.g. B) is to estimate appropriate beta coefficients and the unsystematic component of equity return risk. To do this, a cross-sectional time series is developed from Compustat for firms with various bond ratings for the period 1993–1998. Within each bond rating class the firms are then divided into high

Table 5
Equity return volatility for low and high volatility firms by bond rating category and market volatility

Low volatility firms with bonds rated	Mean beta 1993–1998	Mean firm specific equity return volatility 1993–1998	High volatility firms with bonds rated	Mean beta 1993–1998	Mean firm specific equity return volatility 1993–1998	
Aaa	0.679	0.245	Aaa	0.682	0.317	
Aa	0.649	0.249	Aa	0.757	0.363	
A	0.699	0.222	A	0.864	0.412	
Baa	0.864	0.292	Baa	0.994	0.507	
Ba	1.019	0.425	Ba	1.131	0.729	
			B	1.314	0.727	
			Caa	1.301	0.954	
	Market volatility					
	1993	1994	1995	1996	1997	1998
S&P500 volatility	0.059	0.107	0.050	0.107	0.158	0.230

A cross-sectional time series is developed from Compustat to calculate the average firm's beta by bond rating for the period 1993–1998. Bonds are sorted by bond rating and characteristic lines are estimated to compute the firm's beta and unsystematic (firm specific) risk. The market volatility over the 1993–1998 time period is also displayed.

or low volatility classes. Low volatility firms are defined to be those in the lower third of total equity return volatility. High volatility firms are defined to be the remaining two-thirds of firms. Due to their inherent high volatility B- and Caa-rated firms are not divided into different volatility categories. Characteristic lines are then estimated for each rating and volatility class relating firm's return on equity to the return on a sector equity index. The results are found in Table 5. As bond rating declines, the firm's systematic equity return risk (beta) and unsystematic risk (the annualized root mean square error) increases.

3.3. Simulating an n-variate normal distribution

Fridson et al. (1997) find a positive relation between interest rates and default rates. This is consistent with negative correlations between interest rate changes and equity returns. The historical correlation structure between the change in interest rates, the return on various equity indices, and the U.S. dollar/Japanese yen exchange rate are found in Table 6. For example, the correlation coefficient between changes in the short U.S. Treasury rate and the return on the S&P500 is a negative 0.33. The correlation between interest rates and equity index returns is of course a function of the interest rate sensitivity of the sector.

Table 6
Correlation of the change in interest rates, the return on U.S. industry equity indices, and the yen/U.S. $ exchange rate on a monthly basis from January 1987 to December 1996

	ΔTre-asury	S&-P500	Auto	Bank	Chem	Bldg	Energy	Entmt	Health	Insur	Manuf	OilGas	Paper	Tech	Tele-com	Textile	Util	Yen/ U.S. $
ΔTreasury	1	−0.33	−0.02	−0.23	−0.15	−0.27	−0.23	0.02	−0.18	−0.35	−0.18	−0.03	−0.13	−0.02	−0.26	−0.05	−0.49	0.101
S&P500		1	0.37	0.69	0.66	0.73	0.60	0.56	0.77	0.70	0.71	0.56	0.47	0.71	0.54	0.45	0.57	−0.062
Auto			1	0.41	0.47	0.54	0.22	0.37	0.05	0.19	0.45	0.22	0.43	0.31	0.27	0.15	0.07	0.022
Bank				1	0.52	0.54	0.41	0.49	0.38	0.55	0.49	0.35	0.39	0.51	0.44	0.24	0.32	0.073
Chem					1	0.48	0.51	0.31	0.50	0.32	0.61	0.43	0.64	0.44	0.42	0.30	0.15	−0.054
Bldg						1	0.41	0.57	0.42	0.47	0.71	0.46	0.43	0.51	0.37	0.46	0.39	0.025
Energy							1	0.21	0.33	0.36	0.33	0.46	0.08	0.35	0.25	0.22	0.57	−0.219
Entmt								1	0.39	0.37	0.50	0.34	0.36	0.58	0.23	0.28	0.10	0.033
Health									1	0.52	0.41	0.29	0.33	0.51	0.30	0.39	0.39	−0.014
Insur										1	0.41	0.25	0.19	0.36	0.46	0.21	0.58	0.038
Manuf											1	0.53	0.57	0.61	0.35	0.48	0.24	−0.095
OilGas												1	0.21	0.48	0.20	0.37	0.23	−0.211
Paper													1	0.40	0.37	0.29	0.00	−0.084
Tech														1	0.29	0.32	0.18	−0.126
Telecom															1	−0.01	0.41	−0.003
Textile																1	−0.02	0.007
Util																	1	−0.068
Yen/U.S. $																		1

In the current portfolio risk assessment model, the equity indices and FX rate returns are simulated as stochastic variables correlated with the simulated future risk-free interest rate and interest rate spreads. Hull (1997) describes a procedure for working with an n-variate normal distribution. This procedure requires the specification of correlations between each of the n stochastic variables. Subsequently n independent random samples ε are drawn from standardized normal distributions. With this information the set of correlated random error terms for the n stochastic variables can be calculated. For example, for a bivariate normal distribution,

$$\varepsilon_1 = x_1, \tag{7}$$

$$\varepsilon_2 = \rho x_1 + x_2\sqrt{1-\rho^2}, \tag{8}$$

where x_1, x_2 are independent random samples from standardized normal distributions, ρ the correlation between the two stochastic variables, and $\varepsilon_1, \varepsilon_2$ the required samples from a standardized bivariate normal distribution. It can be shown that the simulated volatilities and correlations for all of the stochastic variables match closely the assumed values.

3.4. Mapping debt ratios into credit ratings

The above simulated equity returns (Eq. (6)) are then used to estimate a distribution of possible future equity market values and debt ratios. The simulated debt ratios are then mapped into credit ratings. This methodology assumes a deterministic relation between the firm's debt ratio and its credit rating. [3] In a contingent claims framework this is equivalent to assuming a constant volatility for the value of the firm.

To implement this method an empirical analysis of the distribution of debt ratio [4] by rating class is performed on all non-financial firms with a Standard & Poor's bond rating tracked by Compustat on a quarterly basis from 1987 to 1998. We segmented the bonds by rating class into two categories, high and low volatility firms, based upon the historical volatility of their equity returns as described above. Debt ratio distributions are then analyzed by rating category and volatility category. The results are found in Table 7. As expected, debt ratio increases as bond rating declines, and high volatility firms have lower average debt ratios. For the Caa–C and Default categories it is noted that there

[3] Blume et al. (1998) suggest that leverage ratios and credit ratings are not constant over time. However, their results are over a longer time frame than simulated in this framework.

[4] Merton (1974) defined leverage ratio as debt over equity. To simplify for comparison purpose, the algebraically equivalent debt over total market capitalization (i.e. debt ratio), defined as [book value of debt/(book value of debt + market value of equity)], is utilized in this study.

Table 7
Debt ratios and bond ratings for firms segmented into low and high volatility firms

Rating	N	Mean	Std. dev.	Max.	Q_3	Median	Q_1	Min.
Low volatility firms								
Aaa	57	0.141	0.127	0.988	0.171	0.121	0.075	0.051
Aa	293	0.241	0.122	0.489	0.334	0.278	0.117	0.011
A	989	0.319	0.132	0.606	0.419	0.350	0.221	0.020
Baa	509	0.341	0.171	0.747	0.463	0.352	0.200	0.018
Ba	723	0.472	0.186	0.943	0.589	0.460	0.333	0.060
High volatility firms								
Aaa	286	0.144	0.145	0.748	0.157	0.101	0.048	0.015
Aa	1067	0.163	0.120	0.690	0.204	0.127	0.077	0.017
A	3646	0.240	0.140	0.821	0.340	0.212	0.131	0.012
Baa	4312	0.319	0.159	0.832	0.431	0.305	0.198	0.011
Ba	3500	0.397	0.207	0.960	0.554	0.386	0.226	0.012
B	3076	0.515	0.235	0.983	0.702	0.525	0.324	0.015
Caa	34	0.729	0.262	0.984	0.931	0.819	0.615	0.117
Def	17	0.779	0.226	0.990	0.940	0.851	0.699	0.127

All non-financial firms with a Standard & Poor's bond rating which Compustat tracked over the period of 1987 to 1998 are identified. Quarterly data on debt ratios and bond rating is obtained. The debt ratio is defined as (book value of short- and long-term debt/(book value of short- and long-term debt + market value of equity)). Due to their inherent high volatility B- and Caa-rated firms are not divided into different volatility categories. For Caa and defaulted companies only the first observation in that category is utilized in the analysis. The descriptive statistics of this analysis are provided.

is very little difference in the distribution of debt to value ratios which are based on the first observation when a firm is reported to have entered these categories. For simulation runs reported later in this study, we assume that debt ratios start at the mid point between the first and third quartiles for the assumed initial credit rating category. Credit ratings are generally assumed to change when simulated debt ratios cross the quartile boundaries. However due to the fact that the distribution of debt to value ratios of Caa–C and defaulted companies is very similar, the debt to value ratio at which firms are assumed to default is set at 0.78. This level is approximately equal to the mean for defaulting firms. Increasing (decreasing) this critical debt to value ratio reduces (increases) simulated bond default rates.

After simulating the bond's future credit rating its value is calculated using the simulated term structure of interest rates appropriate for that risk class. If the bond is simulated to default, the recovery rate on the bond is simulated as a beta distribution [5] with a mean value of 34% and a standard deviation of 25%.

[5] Utilizing a beta distribution allows the recovery rate to fall within 0% and 100% while maintaining the same mean and standard deviation.

If the bond is denominated in a foreign currency then its numeraire currency value is calculated by multiplying the simulated bond value by the simulated foreign exchange rate that by construction is also a correlated stochastic variable. To determine a probability distribution of simulated values, the simulation is run 10,000 times. The distribution of values is then used to determine test statistics and estimates for the 99%, 97.5%, and 95% confidence levels. The final result is a total portfolio risk analysis, which accounts for correlated market and credit risk.

4. Simulation results

In this section, we demonstrate the methodology described previously to undertake various analyses. Unless otherwise noted, the previously stated assumptions are utilized.

4.1. Credit transition matrixes

Utilizing the above models, data, and assumptions a firm's debt ratio and hence credit rating can be simulated over any time step. The results for 10,000 simulations for one-, and three-year time steps are reported in Table 8 for both high and low volatility firms.

Comparisons of the simulated transition matrixes and Moody's historical transition matrixes (Table 2) show many similarities. In each case the most likely event is that the rating stays the same, the next most likely event is that the ratings move up or down by one category. Also the rating transitions become more dispersed as the time step increases (e.g. one-year versus three-year).

Moody's does not distinguish between low and high volatility companies thus there is no direct comparison for historical transition probabilities and the simulated ones for low and high volatility firms. However it is interesting to note that the simulated probabilities of the lower volatility firms staying in their initial rating category are consistently larger than those for the higher volatility firms. Also an average of the simulated transition probabilities for the low and high volatility firms would result in distributions somewhat more dispersed than Moody's historical average. In addition the simulated default rates on Caa–C rated firms are higher than the historical averages. This result is consistent with the volatile conditions which prevailed in the markets during 1998 where the S&P500 had a volatility of 23% versus 20% over the long term. It is also consistent with the high yield on Caa–C securities prevailing at that time (i.e. 21% on ten-year bonds). Over other selected periods (e.g. early to mid 1990s) market volatility and thus simulated default rates would have been lower. Finally it is important to note that the investment grade bonds generally

Table 8
Simulated credit rating transition matrixes

Initial rating	Aaa	Aa	A	Baa	Ba	B	Caa–C	Default
Low volatility firms								
Probability of rating after one year								
Aaa	93.50%	6.50%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa	0.09%	97.67%	2.23%	0.01%	0.00%	0.00%	0.00%	0.00%
A	0.00%	1.51%	94.30%	3.57%	0.62%	0.00%	0.00%	0.00%
Baa	0.00%	0.99%	7.17%	79.41%	12.19%	0.24%	0.00%	0.00%
Ba	0.01%	0.46%	2.40%	7.61%	76.44%	12.13%	0.88%	0.07%
Probability of rating after three years								
Aaa	79.58%	20.33%	0.08%	0.01%	0.00%	0.00%	0.00%	0.00%
Aa	4.08%	81.94%	11.11%	1.63%	1.20%	0.04%	0.00%	0.00%
A	0.09%	9.87%	72.41%	9.14%	8.11%	0.38%	0.00%	0.00%
Baa	0.33%	8.08%	11.42%	52.95%	21.38%	5.42%	0.38%	0.04%
Ba	0.65%	6.41%	6.22%	8.70%	52.21%	16.72%	4.88%	4.21%
High volatility firms								
Probability of rating after one year								
Aaa	80.75%	15.68%	3.56%	0.01%	0.00%	0.00%	0.00%	0.00%
Aa	3.66%	83.51%	12.72%	0.11%	0.00%	0.00%	0.00%	0.00%
A	0.05%	4.38%	82.99%	10.91%	1.66%	0.01%	0.00%	0.00%
Baa	0.01%	1.30%	9.70%	71.39%	15.21%	2.38%	0.01%	0.00%
Ba	0.14%	1.44%	6.47%	4.03%	72.22%	14.12%	1.22%	0.36%
B	0.00%	0.25%	0.80%	1.15%	9.15%	78.15%	6.60%	3.90%
Caa–C	0.02%	0.34%	1.18%	1.14%	5.80%	9.17%	58.80%	23.55%
Probability of rating after three years								
Aaa	67.69%	17.54%	14.07%	0.67%	0.03%	0.00%	0.00%	0.00%
Aa	13.37%	60.48%	22.79%	2.85%	0.51%	0.00%	0.00%	0.00%
A	3.05%	10.90%	61.05%	14.90%	8.36%	1.68%	0.04%	0.02%
Baa	1.99%	6.71%	12.42%	50.77%	17.71%	9.51%	0.64%	0.25%
Ba	2.87%	5.84%	9.11%	3.98%	55.89%	15.53%	2.81%	3.97%
B	1.16%	2.49%	5.05%	2.52%	10.66%	51.81%	5.18%	21.13%
Caa–C	1.34%	2.37%	3.94%	1.74%	7.18%	6.94%	28.95%	47.54%

Utilizing a CCA framework, simulated credit rating transition matrixes are estimated for low and high volatility firms by bond rating category. The transition matrices are a function of the volatility of the equity market indices (e.g., S&P500), and the firm's risk, the unsystematic risk, debt ratio, and dividend yield. The volatility of the equity market index (0.23) is estimated for the year 1998. The firm specific parameters are estimated over the 1993–1998 period.

had a zero or very low simulated default rate while Moody's shows some small percentage. This is a limitation of the proposed methodology. Possible explanations for these differences include inaccuracies in the proposed model or its estimated parameters, non-normal equity return distributions including infrequent catastrophic losses, delays in bond rating changes by rating agencies, actions by some companies to maintain a target bond rating by adjusting investment and financing strategies, a dispersion of firm characteristics not captured by the standard assumptions used in the analysis, and occasional changes in firms' target capital structures (e.g. leveraged buyouts). Of course simulated default rates can be increased (decreased) by lowering (raising) the debt to value ratio at which default is assumed to occur.

Overall it is concluded that the model produces transition probabilities similar to the reported historical transitions. It is important to note that each firm's bond rating is related systematically to the returns on the equity index for the sector in which it operates. Thus the model captures the impact of macro financial market volatility. Further because the equity indices for various sectors are correlated with each other the simulation captures correlated credit risk for bond portfolios.

4.2. Bond valuation tests

To test the ability of the model to value bonds, comparisons are made between analytical and simulated coupon bond prices. Bond prices with a maturity of ten years are calculated from the known typical yield curves for each rating class as of December 1998. The values of the coupon bonds are then simulated out one, and three years ($t = 1, 3$) and discounted back at the average simulated risk-free rate. For the model to be arbitrage-free, the known value at $t = 0$ (\$100) should equal the simulated value (at $t = 1, 3$) discounted back to $t = 0$ at the risk-free rate. A deviation between the known value and the simulated value implies a mispricing in a risk-neutral valuation framework.

Using the standard simulation assumptions, the bond valuation tests are performed on bonds with assumed initial credit ratings of Aaa through Caa. The results are found in Table 9. The estimated error represents the difference between the mean simulated values and the analytical solutions. The results suggest that the simulation models are reasonably accurate for bond rating categories Aaa through Baa for both one- and three-year time steps, where the models produce close to arbitrage-free estimates in most cases with no error exceeding 2%. For non-investment grade bonds the simulated prices are somewhat higher than the analytical values, particularly so for the three-year time step. The finding of a premium in a risk neutral valuation framework for non-investment grade bonds is consistent with Fons (1987), Altman (1989), and Jarrow et al. (1997). It should also be noted that December 1998 is a time of wide credit spreads which is consistent with a liquidity premium for holding such securities. Finally if the objective is to produce arbitrage-free values for a particular type of bond (e.g. B-rated) then it could easily be accomplished by adjusting the debt to value ratio at which firms are assumed to default.

Table 9
Bond valuation tests

	Aaa	Aa	A	Baa	Ba	B	Caa–C
Simulated price at t = 1 discounted back at the risk-free rate							
High volatility firms							
Mean value	997.15	1001.83	999.09	980.25	1037.15	1012.21	1025.80
Std. dev.	42.10	40.24	50.36	90.73	119.97	199.83	490.15
% pricing error	−0.29	0.18	−0.09	−1.98	3.72	1.22	2.58
Low volatility firms							
Mean value	1000.88	1002.72	1002.77	989.48	1035.12	n.a.	n.a.
Std. dev.	40.09	39.68	44.91	77.92	105.69	n.a.	n.a.
% pricing error	0.09	0.27	0.28	−1.05	3.51	n.a.	n.a.
Simulated price at t = 3 discounted back at the risk-free rate							
High volatility firms							
Mean value	1004.49	1016.71	1007.88	994.89	1118.79	1052.70	1086.43
Std. dev.	75.29	79.29	103.50	134.35	188.04	366.56	694.47
% pricing error	0.45	1.67	0.79	−0.51	11.88	5.27	8.64
Low volatility firms							
Mean value	1008.46	1016.54	1009.54	998.03	1104.48	n.a	n.a.
Std. dev.	74.10	81.15	102.48	126.84	205.10	n.a.	n.a.
% pricing error	0.85	1.65	0.95	−0.20	10.45	n.a.	n.a.

The value represents the value of a ten-year coupon bond simulated out one and three years in a risk-neutral framework and then discounted back at the risk-free rate ($V_0 = V_t e^{-rt}$). The simulation output contains the mean value and the standard deviation of the simulated values. The estimated error represents the over- or under-valuation of the simulated mean compared to the initial market value of $1000.

4.3. Risk analysis

After examining the transition probabilities and valuation for a single bond, we next examine the model's ability to analyze integrated market and credit risk for a portfolio of bonds. A principal advantage of using a simulation model in the portfolio analysis is the ability to relate financial environment volatility (i.e. equity index volatility) to firm specific credit risk. Further since equity index returns are correlated with other stochastic variables (e.g. interest rates) correlated market and credit risk for a portfolio can be estimated. For example a bond portfolio that is highly concentrated in one industry would have less credit risk diversification, while a bond portfolio which is diversified across a large number of industries will have diversified credit risk to a greater extent. Also during periods of high market volatility simulated market and credit risk both increase.

The risk analysis demonstration will first focus on a single bond and subsequently consider portfolios of bonds. The value of the bond is simulated at the end of the time period and includes the last coupon payment. The risk

Table 10
Simulated VaR measures for a B-rated bond

One B-rated bond facing various risks					
Interest rate risk	Yes	Yes	No	Yes	Yes
Interest rate spread risk	No	Yes	No	Yes	Yes
Credit risk	No	No	Yes	Yes	Yes
FX risk	No	No	No	No	Yes
Mean value	108,866	109,081	104,148	104,225	104,426
Std. dev.	3,215	6,450	22,561	23,433	25,869
Change in std. dev.	n.a.	3,235	16,111	872	2,436
Maximum value	121,238	126,769	153,464	164,057	198,049
Minimum value	98,037	72,129	29	21	18
VaR confidence levels					
99% level	101,718	90,096	9,130	8,844	9,187
97.5% level	102,790	94,406	29,164	29,791	29,427
95% level	103,723	97,180	60,544	56,736	53,230

VaR measures are simulated for a B-rated bond with an initial value of $100,000 at one-year time step. The value of the bond is equal to the price at $t = 1$ plus the coupon payment if the bond did not default.

analysis for a single ten-year B-rated bond at a one-year time step is found in Table 10. Initially the risk analysis is performed with only interest rate risk. Under this assumption the mean simulated value of the bond is $108,866 with a standard deviation of $3,215 and 95% confidence level of $103,723. The inclusion of interest rate spread risk has little impact on the mean value ($109,081) however the standard deviation doubles to $6,450 and the 95% confidence level declines to $97,180. Thus spread risk is clearly a significant risk factor. For example spread risk is said to have caused large losses for Long-Term Capital Management.

Next a risk analysis is performed on credit risk only. Credit risk reduces the mean simulated value to $104,148 (due to credit downgrades and default losses), sharply increases the standard deviation to $22,561, and sharply reduces the 95% confidence level to $60,544. In the extreme the minimum value of the bond falls to $29 reflecting the possibility of default with minimal recovery. The simulated standard deviation for bond value resulting from credit risk alone is somewhat higher than that calculated in Table 3 using a standard credit risk analysis ($17,412 for a $100,000 initial value). This difference is explained by the fact that the simulated probabilities for higher volatility firms migrating out of the B-rating category at the end of 1998 are somewhat larger than Moody's average historical credit transition probabilities. Also the penalty for downgrading to Caa is large due to the unusually high yield on Caa bonds (21%).

The inclusion of interest rate risk, and spread risk along with credit risk has little impact on the mean value of the bond ($104,225 versus $104,148), however it marginally increases the standard deviation by $872 ($23,433 versus

$22,561) and reduces the 95% confidence interval from $60,544 to $56,736. This small increase in the standard deviation of bond value ($872) suggests that for the current simulation the covariance between total interest rate risk (risk-free plus spreads) and credit risk is small. The low covariance between total interest rate and credit risk in this case is a function of several factors. First B-rated firm's have a high level of firm specific equity return volatility (72.7%) relative to systematic equity index return volatility (23%). Second, the assumed correlation between interest rate changes and equity index returns is only −0.33. Third, the level of volatility for risk-free interest rates is relatively low. In other circumstances (time periods, countries) these relationships change and the volatilities and correlations between these various risk factors increase. In any event, accounting for correlations correctly is always important if overall risk levels are to be calculated appropriately.

The inclusion of FX risk has little impact on the mean value of the bond ($104,426) however it further increases the standard deviation ($25,869), and reduces the 95% confidence level ($53,230). Thus FX risk is, as expected, also a significant risk factor.

To perform portfolio risk analyses, we form portfolios of 1, 2, 5, 7, 10, 15, 20, 24, and 100 B-rated bonds drawn from up to 24 economic sectors. The results are found in Table 11. All bonds are assumed to have a ten-year maturity and be non-callable. As possible each bond added to the portfolio is from a different industry with equity index correlations found in Table 6. Hence, our resulting estimates are for the maximum diversification available for the number of bonds in the portfolio. The ending value of the portfolio is simulated out one year, 10,000 times. For comparison purposes we include one portfolio of 24 bonds that faces FX risk as well. We also include portfolios of 24 and 100 bonds drawn from a single industry.

As can be seen from Table 11, as the number of bonds included in the portfolio increases there is little change in the mean portfolio value (i.e. $104,200 to $104,500). More importantly from a risk analysis perspective, as the number of bonds in the portfolio increases, the standard deviation decreases from $23,433 to $8,102 for portfolios with 100 bonds drawn from 24 sectors, or $9,518 for portfolios with 100 bonds drawn from one sector. Further the minimum value and confidence levels increase (e.g. 95% confidence level increases from $56,625 to $90,226 for portfolios with bonds drawn from separate sectors, or $87,656 for portfolios with bonds drawn from one sector). Firm and sector diversification clearly pays. However during periods when correlations increase and systematic equity return risk increases relative to firm specific risk such diversification benefits may prove to be less than expected.

As discussed previously interest rate and spread risk taken alone produce a portfolio standard deviation of $6,450. With 24 bonds credit risk taken alone produced a portfolio standard deviation of $6,810. With 24 bonds interest rate, spread, and credit risk produced a standard deviation of $8,878. As would be

Table 11
Portfolio risk analysis

Distribution of simulated values for a $100,000 initial value portfolio of B-rated Bonds at a one-year time step facing various risks

Risk included in analysis													
Interest rate risk	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes
Interest rate spread risk	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes
Credit risk	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
FX risk	No	No	No	No	No	No	No	No	No	Yes	No	No	No
Number of sectors	1	2	5	7	10	15	20	24	24	24	1	1	24
Number of bonds	1	2	5	7	10	15	20	24	24	24	24	100	100
Mean value	104,225	104,373	104,425	104,291	104,320	104,285	104,232	104,279	104,093	104,282	104,464	104,547	104,419
Std. dev.	23,433	17,860	11,948	10,733	9,686	9,026	8,995	8,878	6,810	13,825	9,853	9,518	8,102
Change in std. dev.	n.a.	−5,573	−5,912	−1,215	−1,047	−660	−31	−117	−2,068	7,015	−3,972	−335	−1,416
Maximum value	164,057	156,231	140,689	134,290	133,147	132,428	131,366	130,604	125,907	166,716	133,943	137,339	131,448
Minimum value	21	3,591	48,410	50,843	51,979	64,674	64,684	66,434	69,835	51,741	45,175	53,956	58,754
VaR confidence levels													
99% level	8,844	46,461	71,385	76,005	78,283	80,545	80,436	80,786	84,826	73,479	78,108	79,217	83,625
97.5% level	29,791	57,522	77,336	80,258	83,542	85,323	84,827	85,032	88,761	78,729	82,652	83,950	87,192
95% level	56,736	65,747	82,185	84,897	87,131	88,379	88,312	88,766	91,553	82,159	86,407	87,656	90,226

To perform portfolio risk analyses, we form portfolios of 1, 2, 5, 7, 10, 15, 20, 24, and 100 B-rated bonds. All bonds are assumed to have a ten-year maturity, are non-callable, and face interest rate, interest rate spread, and credit risk. We also include one portfolio of 24 bonds that faces FX risk as well. For portfolios drawn from more than one sector the correlations are based upon the historical estimates found in Table 6. Hence, our resulting estimates are for the maximum diversification available for the number of bonds in the portfolio. The ending value of the portfolio is simulated out one year, 10,000 times. The ending value includes the value of the bonds plus the coupon payment. The mean, standard deviation, maximum, and minimum simulated portfolio value, and the 99%, 97.5%, and 95% confidence levels are the resulting output statistics from the model.

expected interest rate, spread, and credit risk are clearly not additive. The importance of integrated risk analysis that accounts for the correlations in these risk factors is apparent.

The results also have implications for the number of bonds necessary to diversify away credit risk. There is a significant reduction in risk as bonds are added to the portfolio. However, the gains from diversification are relatively small after 15 bonds. The implication is that a bond investor diversifies away much of the unsystematic risk with 15 bonds in 15 different industries. However, larger portfolios of 100 bonds drawn from a variety of industries do have somewhat better risk characteristics.

Adding FX risk again has little impact on the mean value of the 24-bond portfolio ($104,282). However it substantially increases portfolio standard deviation ($13,825 versus $8,878) and reduces the 95% confidence level ($82,159 versus $88,766).

4.4. Comparison of simulated risk analysis versus historical risk analysis

To check the validity of the simulation model, we compute common risk measures for a B-rated bond and portfolios of B-rated bonds using historical data. We use two different methodologies to estimate historical risk measures. We then examine the differences between the simulated and historical risk measures.

First, to compare our simulation results for a portfolio of single B bonds with historical risk measures, we calculated the one-year holding period returns on a monthly basis from 1987 through 1998 for the Lehman Brothers B-rated long-term bond index. We then determined the 99%, 97.5%, and 95% confidence levels for a $100,000 investment over a one-year time frame which are $87,416, $89,855, and $94,906 respectively. The historical results are then compared to the simulation results found in Table 11 for a 100-bond portfolio with interest rate, spread risk, and credit risk. The simulated confidence levels are $83,625, $87,192, and $90,226 at the 99%, 97.5%, and 95% confidence levels respectively.

This moderately higher risk level in the simulation results is appropriate for two reasons. First, the simulations are for 100 bonds as compared to the larger portfolio of bonds in the index. Second and more importantly, the Lehman Brothers index is refreshed every month with single B bonds. Thus if a bond is downgraded, it would effect the index return for that month, but the bond is then removed from the index for the following month. Hence, the bond index only catches one downgrade, while in the simulation model multiple downgrades (including defaults) in a single year are possible. This comparison between simulated results and analytical results suggests that the simulation model produces results consistent with historical measures.

Table 12
Historical versus simulated VaR analysis for B-rated bond portfolios

Distribution of values for a $100,000 initial value for a single bonds and a portfolio of B-rated Bonds at a one-year time step facing various risks

Risk included in analysis						
Interest rate risk	Yes	Yes	Yes	Yes	Yes	Yes
Interest rate spread risk	Yes	Yes	Yes	Yes	Yes	Yes
Credit risk	Yes	Yes	Yes	Yes	Yes	Yes
FX risk	No	No	No	No	No	No
Type of analysis	Historical	Simulated	Difference	Historical	Simulated	Difference
Number of bonds	1	1	1	15	15	15
Mean value	102,034	104,225	(2,191)	102,052	104,285	(22,330)
Std. dev.	20,370	23,433	(3,063)	8,198	9,026	(828)
Change in std. dev.	n.a.	n.a.	n.a.	(12,172)	(14,407)	2,235
Maximum value	191,265	164,057	27,208	114,887	132,428	(17,541)
Minimum value	3,440	21	3,418	74,746	64,674	10,072
VaR confidence levels						
99% level	23,494	8,844	14,650	80,147	80,545	−398
97.5% level	36,586	29,791	6,795	88,248	85,323	2,925
95% level	67,250	56,736	10,514	89,765	88,379	1,386

Historical values reflect total returns (coupon plus price change) for actual B-rated bonds over the 1998–1999 period. We perform a historical analysis based on 625 B-rated bonds, which are identified from data available from Chase Manhattan Bank and Moody's bond record as of 12/31/97. Simulated values are based on the model calculated for December 1998.

Second, we perform a historical analysis, Table 12, based on 625 B-rated bonds, which are identified from data available from Chase Manhattan Bank and Moody's bond record as of 12/31/97. These bonds are divided randomly into 12 groups. Each of these groups of bonds is assigned to a month (i.e. January to December). Then the annual total return on each bond and portfolios of bonds are calculated (e.g. January 1998 to January 1999, ..., December 1998 to December 1999). Total return is defined as the price change of the bond plus the coupon. If the bond defaulted, then it is assumed that the coupon is not received.

The information in Table 12 indicates that the simulation model provides a VaR analysis for B-rated bonds reasonably similar to historical levels. For a single B-rated bond, the historical standard deviation of the value of the bond is $20,370 versus $23,433 simulated. The historical maximum and minimum are $191,265 and $3,440 respectively versus $164,057 and $21 simulated. The 99% and 95% VaR points on a historical basis are $23,494 and $67,250 versus $8,844 and $56,736 simulated.

For B-rated portfolios of approximately 15 bonds the historical and simulated value at risk analyses are remarkably close. Table 12 indicates that for a portfolio of approximately 15 bonds, the historical standard deviation of the value of the portfolio is \$8,198 versus \$9,026 simulated. The historical maximum and minimum are \$114,887 and \$74,746 versus \$132,428 and \$64,674 simulated. The 99% and 95% value at risk points on a historical basis are \$80,147 and \$89,765 versus \$80,545 and \$88,379 simulated. More extensive comparisons of historical and simulated value at risk analyses are an important area for future work.

5. Conclusion

Current portfolio risk estimation methodologies calculate market and credit risk in separate analyses. There is no reliable method for combining these risk measures into one overall portfolio risk assessment. Such risk estimation errors have significant implications for many types of financial decisions including financial institution capital adequacy requirements.

This paper provides a methodology to assess correlated market and credit risk. These risks are jointly estimated by simulating both the future financial environment in which financial instruments will be valued and the credit rating of specific firms. The fundamental basis of this methodology is a reduced form of the CCA proposed by Merton (1974). Given the number of stochastic variables and the complexity of the relationships no closed form analytical solution for calculating the needed risk measures is available. Thus the analysis is undertaken with a simulation model.

The viability of the model is tested by comparing simulated credit rating transition probabilities to historical transition probabilities, simulated and analytical bond prices, and simulated and historical portfolio VaR analyses. Simulated credit rating transition probabilities are shown to reasonably approximate historical patterns, but the model does underestimate the frequency of large jumps in credit ratings over a one-year time frame. The bond valuation tests show that the model works better for investment grade than non-investment grade bonds. However the overpricing of non-investment grade bonds is consistent with the findings of risk and liquidity premiums by other authors.

The risk assessment methodology applied to a single bond demonstrates that while all four risk factors (interest rate, spread, credit, and FX risk) are important the most important for non-investment grade bonds is credit risk. Thus a crucial data requirement for any risk assessment is the credit quality of the security.

The portfolio analysis capabilities of the model highlight the importance of diversification of credit risk across a number of firms and sectors of the economy. Simulated and historical VaR risk measures for B-rated bond portfolios are shown to be very similar.

The model can be extended to deal with other types of financial instruments such as mortgages, variable rate loans, equities, and derivatives. Potential applications extend beyond valuing and modeling bond portfolios to undertaking financial institution risk assessments, evaluating alternative hedging strategies, assessing capital adequacy, and undertaking financial system systemic risk analyses.

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